Magnetic control of convection in nonconducting diamagnetic fluids

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Inhomogeneous magnetic fields exert a body force on electrically nonconducting, diamagnetic fluids. This force can be used to compensate for gravity and to control convection. The field effect on convection is represented by a dimensionless vector parameter $\mathbf{R}_m=(\mu_0\alpha\chi_0 d^3\Delta T/\rho_0\nu D_T)(\mathbf{H}\cdot\boldsymbol{\nabla}\mathbf{H})^{\text{ext}}_{\mathbf{r}=\mathbf{0}}$, which measures the relative strength of the induced magnetic buoyancy force due to the applied field gradient. The vertical component of this parameter competes with the gravitational buoyancy effect and a critical relationship between this component and the Rayleigh number is identified for the onset of convection. Magnetically driven convection should be observable even in pure water using current technology. [S1063-651X(98)10210-6]

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I. INTRODUCTION

In recent papers we have explored the effect of static magnetic fields on the stability of a horizontal layer of electrically nonconducting paramagnetic fluid bounded above and below by no-slip surfaces maintained at different temperatures. When the imposed magnetic field is uniform and the layer is heated from below, convection begins at a lower Rayleigh number than in the nonmagnetic (Rayleigh-Bénard) case, with the deviation increasing with the absolute value of the vertical field component [1]. The motion begins as two-dimensional rolls whose axes are parallel to the horizontal component of the imposed field. For an imposed inhomogeneous magnetic field, we demonstrated the existence of an additional magnetic body force whose vertical component also competes with gravitational buoyancy [2,3]. This component of the force can enhance or suppress Rayleigh-Bénard convection depending on its sign. These predictions are in agreement with recent experiments [4,5]. Of particular note is the fact that this force can drive convection even in the absence of gravity.

In this Brief Report we extend our analysis to diamagnetic fluids. This is an important extension because the class of diamagnetic fluids is much larger than the class of paramagnetic fluids. It includes many of the most practically important fluids such as water [liquid (*l*) and gas (*g*)] and nitrogen (*l* and *g*). Some other diamagnetic fluids are carbon monoxide, carbon dioxide, hydrogen (*g*), chlorine (*g*), ammonia (*g*), sulfur (*l*), sulfuric acid, and all of the noble gases. Virtually all organic compounds are diamagnetic including methane, benzene, ethyl alcohol, and glycerol [6]. Unlike paramagnetic fluids, diamagnetic fluids contain atoms or molecules that have no intrinsic magnetic moment. When a static magnetic field is applied to these fluids, the change of the field induces a small additional current inside each atom or molecule. The resulting induced magnetic moments are directed opposite to the field according to Lenz's law. Thus nonuniform fields repel these induced moments away from the high-field regions, which gives rise to the repulsive magnetic body force. This force is called the Kelvin force. Although the physics of diamagnetism has been well known for many years, the implications are easily forgotten because the magnitude of diamagnetic susceptibilities is such that the Kelvin force is usually negligible. However, our analysis will show that this small effect can now be utilized to control convection even in pure water using current magnetic technology.

In contrast to paramagnetic fluids [2,3], the susceptibility of diamagnetic fluids is not an explicit function of temperature. This can lead to the erroneous conclusion that there can be no interaction between the Kelvin force and the temperature field. However, a nonuniform temperature field can give rise to a nonuniform Kelvin force in a manner analogous to the way in which it can produce gravitational buoyancy. Although the phenomenon now seems obvious, it was apparently not obvious to the pioneers of magnetothermal convection because it was not discussed for many years. The early papers of Carruthers and Wolfe [7] and Clark and Honeywell [8] make no mention of magnetothermal convection in diamagnetic fluids. Braithwaite, Beaugnon, and Tournier [4] and Beaugnon *et al.* [5], who experimentally demonstrated magnetothermal convection in a paramagnetic liquid, made no mention of experiments on diamagnetic fluids, even though it was within the capability of their apparatus, as we shall show. Our first paper on this topic [9] does mention the phenomenon, but it focuses on paramagnetic fluids. Not until Houston and Tillotson [10] can we find any detailed discussion of magnetothermal convection in diamagnetic fluids in the literature.

In this Brief Report we examine the conditions under which the Kelvin force can be used to balance the effect of gravity in terrestrial experiments and therefore to control convection in diamagnetic fluids. In a microgravity environment where the gravitational effect can be neglected, the Kelvin force provides a primary body force on electrically nonconducting diamagnetic fluids.

II. GOVERNING EQUATIONS

The quantitative description of the Kelvin force per unit volume is $\mathbf{f}_m=\mu_0(\mathbf{M}\cdot\boldsymbol{\nabla})\mathbf{H}$. Here μ_0 is the permeability of free space, $\mathbf{M}$ is the magnetization (the magnetic moment per unit volume), and $\mathbf{H}$ is the local magnetic field. For diamagnetic fluids $\mathbf{M}=\chi\mathbf{H}$, where χ is the volumetric magnetic susceptibility and is negative. Under ordinary conditions, $\chi \cong -10^{-5}$ for liquids and $\chi \cong -10^{-8}$ for gases in SI units. Diamagnetic susceptibilities depend only on the number of atoms or molecules per unit volume. This fact can be expressed as $\chi=\chi_m\rho$, where ρ is the mass per unit volume and χ_m is the susceptibility per unit mass, a negative constant characteristic of the fluid. Combining these results gives

$$\mathbf{f}_m=\mu_0\chi(\mathbf{H}\cdot\nabla)\mathbf{H}=\mu_0\chi_m\rho\nabla H^2/2. \tag{1}$$

In this form it is clear that the Kelvin force on a diamagnetic fluid is directed away from high-magnetic-field regions and is proportional to ∇H^2. An alternative arrangement $\mathbf{f}_m=\rho[\mu_0\chi_m\nabla H^2/2]=\rho\mathbf{g}_{\mathrm{eff}}$ reveals that the Kelvin force on a diamagnetic fluid can be conceptualized as a gravity force whose direction is determined by ∇H^2. To balance gravity, we require $\mathbf{g}_{\mathrm{eff}}=-\mathbf{g}$, where $\mathbf{g}$ is the acceleration of gravity. This equation yields the required field-field gradient product to levitate diamagnetic fluids on the Earth. For example, for liquid or gaseous water ($\chi_m=9.06\times10^{-9}$ m^3/kg), this requires $|B\nabla B|\simeq\mu_0^2|H\nabla H|=1.36\times10^3$ T^2/m. Here the magnetic induction $\mathbf{B}\equiv\mu_0(\mathbf{M}+\mathbf{H})\simeq\mu_0\mathbf{H}$ because of the small magnetic susceptibility χ for diamagnetic fluids. In a recent experiment, Beaugnon and Tournier [11] have successfully levitated various diamagnetic solids and liquids using a strong nonuniform static magnetic field. Water was levitated by a field with 2961 T^2/m$<|B\nabla B|<$3097 T^2/m, which is higher than expected. They attribute this discrepancy to the wetting effects in their apparatus.

The possibility of magnetothermal convection in diamagnetic fluids arises when the density is a function of temperature. In the simplest case this can be expressed as

$$\rho=\rho_0[1-\alpha(T-T_0)], \tag{2}$$

where α is the coefficient of thermal expansion, a fluid property that is usually positive, T is the temperature, and the subscript 0 denotes a reference state. A temperature difference $\delta T=T-T_0$ then creates a magnetic buoyancy force per unit volume

$$\delta\mathbf{f}_m=\delta\rho\mathbf{g}_{\mathrm{eff}}=-\rho_0\alpha\,\delta T[\mu_0\chi_m\nabla H^2/2]. \tag{3}$$

Under appropriate conditions, this force can drive convection, similar to gravitational buoyancy-driven convection.

To study magnetically controlled convection in electrically nonconducting diamagnetic fluids, we consider an incompressible horizontal layer of such fluid heated on either top or bottom in the presence of an external nonuniform magnetic field. We choose our coordinate system by defining $|z|<d/2$ with $\hat{\mathbf{z}}$ pointing up, where d is the layer thickness. We assume that the field satisfies $\mathbf{H}^{\mathrm{ext}}=\mathbf{H}_0+(\mathbf{r}\cdot\nabla)\mathbf{H}^{\mathrm{ext}}$, where $\mathbf{r}=x\hat{\mathbf{x}}+y\hat{\mathbf{y}}+z\hat{\mathbf{z}}$ is the position vector. Here the vector $\mathbf{H}_0$ is the field at the center of the layer and the field gradient $\nabla\mathbf{H}^{\mathrm{ext}}$ is a constant tensor. Maxwell's equations require this tensor to be symmetric and traceless.

The fluid flow is governed by the Navier-Stokes equations in addition to Maxwell's equations for the magnetic field $\mathbf{H}$ and magnetic induction $\mathbf{B}$. Under the Oberbeck-Boussinesq approximation, which allows density variations only in the large gravity term of the Navier-Stokes equations, we can derive the dimensionless governing equations for the convective flow for nonconducting diamagnetic fluids similar to that for nonconducting paramagnetic fluids [3],

$$\frac{1}{\mathrm{Pr}}\left(\frac{\partial\mathbf{v}}{\partial t}+\mathbf{v}\cdot\nabla\mathbf{v}\right)=-\nabla p+(R\hat{\mathbf{z}}-\mathbf{R}_m)\theta + K\sin^2\phi\,\theta\hat{\mathbf{z}}+K(z-\theta)\hat{\mathbf{H}}_0\cdot\nabla\mathbf{h}+\nabla^2\mathbf{v}, \tag{4}$$

$$\left(\frac{\partial\theta}{\partial t}+\mathbf{v}\cdot\nabla\theta-\hat{\mathbf{z}}\cdot\mathbf{v}\right)=\nabla^2\theta+\Phi, \tag{5}$$

$$\nabla\cdot\mathbf{h}-\hat{\mathbf{H}}_0\cdot\nabla\theta=0, \tag{6}$$

$$\nabla\cdot\mathbf{v}=0. \tag{7}$$

Here $\mathbf{v}$, p, θ, and $\mathbf{h}$ represent the respective departures of velocity, pressure, temperature, and magnetic field from the static thermal conduction state. In these equations $\hat{\mathbf{H}}_0=\mathbf{H}_0/H_0$ is the unit vector in the $\mathbf{H}_0$ direction, ϕ the angle between $\mathbf{H}_0$ and the horizontal, and Φ the viscous dissipation. Equation (4) involves the Prandtl number $\mathrm{Pr}=\nu/D_T$, the Rayleigh number $R=\alpha g d^3\Delta T/\nu D_T$, the Kelvin number

$$K=\frac{\mu_0\alpha^2\chi_0^2\Delta T^2 d^2 H_0^2}{\rho_0\nu D_T}, \tag{8}$$

and the vector control parameter

$$\mathbf{R}_m=\frac{\mu_0\alpha\chi_0 d^3\Delta T}{\rho_0\nu D_T}\,(\mathbf{H}\cdot\nabla\mathbf{H})^{\mathrm{ext}}_{\mathbf{r}=\mathbf{0}}, \tag{9}$$

where ν is the kinematic viscosity, D_T the thermal diffusivity, T_0 the average temperature of the layer, ΔT the temperature difference between the bottom and the top, χ_0 the susceptibility at T_0, and ρ_0 the density at T_0.

III. RESULTS AND IMPLICATIONS FOR EXPERIMENTS

The Rayleigh number R in Eq. (4) measures the strength of gravitational buoyancy relative to dissipation. In the absence of magnetic fields, the thermal convective instability in a fluid layer heated from below is determined by this parameter R and Rayleigh-Bénard convection sets in for $R>R_c\approx1708$. In the presence of a uniform magnetic field ($K\neq0$ but $\mathbf{R}_m=\mathbf{0}$), the magnetic effect on convection is determined by the Kelvin number K and the angle ϕ. For ordinary diamagnetic fluids such as water, our linear stability analysis shows that the difference for the marginal state due to the magnetic effect is less than 0.1% for a field up to 30 T and therefore the uniform field effect on convection in these fluids might be negligible.

The vector parameter $\mathbf{R}_m$ in Eq. (4) measures the relative strength of the magnetic buoyancy force [Eq. (3)] due to the applied field gradient. Since this parameter is the only one containing the external field gradient $\nabla\mathbf{H}^{\mathrm{ext}}$ in the governing equations (4)–(7), the effect of the field gradient on convection in a diamagnetic fluid layer is completely characterized

TABLE I. Summary of results.

Case	ΔT	$H\frac{\partial H}{\partial z}$	$\mathbf{f}_m$	R_m	Result
1	+	+	↓	−	Field promotes Rayleigh-Bénard convection
2	+	−	↑	+	Field inhibits Rayleigh-Bénard convection
3	−	+	↓	+	No convection
4	−	−	↑	−	Magnetothermal convection possible

by this vector parameter. The combination of the vertical component of $\mathbf{R}_m$ with R in Eq. (4) shows that the gravitational effect on the convective flow can be balanced by this component of $\mathbf{R}_m$. Therefore, convection in electrically nonconducting diamagnetic fluids can be controlled by an inhomogeneous magnetic field.

It is instructive to investigate the magnetothermal convective instability of diamagnetic fluids in a magnetic field $\mathbf{H}^{\text{ext}}=H_0\hat{\mathbf{z}}-H_1x\hat{\mathbf{x}}-H_1y\hat{\mathbf{y}}+2H_1z\hat{\mathbf{z}}$, where H_0 and H_1 are constants. A solenoid whose axis coincides with the z axis produces such a field approximately in the central area near the end of the coil. The parameters H_0 and H_1 are determined by the geometrical properties of the solenoid and the electric current. This field yields the vector parameter $\mathbf{R}_m=R_m\hat{\mathbf{z}}$, where

$$R_m=\frac{\mu_0\alpha\chi_0 d^3\Delta T}{\rho_0\nu D_T}\left(H\,\frac{\partial H}{\partial z}\right)^{\text{ext}}_{\mathbf{r}=\mathbf{0}}=\frac{2\mu_0\alpha\chi_0 d^3\Delta T H_0 H_1}{\rho_0\nu D_T}.$$

Under rigid (no-slip) boundary conditions, the linear stability analysis yields the critical condition

$$\frac{\alpha d^3\Delta T_c}{\nu D_T}\left[g-\frac{\mu_0\chi_0}{\rho_0}\left(H\,\frac{\partial H}{\partial z}\right)^{\text{ext}}_{\mathbf{r}=\mathbf{0}}\right]=R_c. \qquad (10)$$

Convection sets in for $\Delta T>\Delta T_c$.

Equation (10) shows that the effect of the magnetic field on the convective instability in diamagnetic fluids depends on the sign of the parameter R_m. A negative R_m will enhance this instability, but a positive R_m will suppress the instability. For diamagnetic fluids, the magnetic susceptibility is the only negative material property. The sign of R_m is determined by the signs of ΔT and $(H\partial H/\partial z)^{\text{ext}}_{\mathbf{r}=\mathbf{0}}$. The four possible cases are summarized in Table I. In cases 1 and 2, the temperature difference ΔT is positive, indicating that the layer is heated from below. Gravity induces a gravitational buoyancy force that tends to destabilize the layer. In the absence of magnetic fields, Rayleigh-Bénard convection sets in for $R>R_c$. In the presence of the field, we see that a downward Kelvin force enhances this convection (case 1), whereas an upward Kelvin force inhibits the convection (case 2). To suppress the convection completely in water requires $|(H\partial H/\partial z)^{\text{ext}}_{\mathbf{r}=\mathbf{0}}|>1.36\times10^3$ T^2/m. As this value has already been exceeded [11], an experimental test of the present theory is now feasible. In cases 3 and 4, the layer is heated from above and gravity tends to stabilize the layer. Table I shows that a downward Kelvin force enhances this stability (case 3) and there is no convection. However, an upward Kelvin force induces a magnetic buoyancy force that tends to destabilize the layer (case 4). When $|(H\partial H/\partial z)^{\text{ext}}_{\mathbf{r}=\mathbf{0}}|>1.36\times10^3$ T^2/m, this destabilizing Kelvin force overwhelms the stabilizing gravitational force and magnetothermal convection sets in for $\Delta T>\Delta T_c$. Experiments are solicited to test these predictions.

IV. CONCLUSION

In conclusion, thermal convection in electrically nonconducting diamagnetic fluids can be controlled by an external inhomogeneous magnetic field through the vector parameter $\mathbf{R}_m$. The inhomogeneous field exerts a magnetic body force on these fluids and this force can balance the gravitational body force in terrestrial experiments. This magnetic-field-induced body force can be utilized to control the flow of diamagnetic fluids in a microgravity environment with possible applications in mixing, heat transfer, and materials processing.

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